Character varieties and Kähler groups

Marco Spinaci

Max Planck Institut für Mathematik

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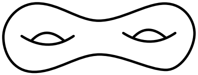
$X =$ 

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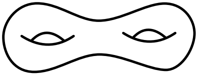
$\Gamma = \pi_1(X)$ → $X =$ 

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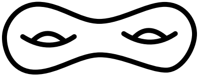
$\boldsymbol{X} =$ 

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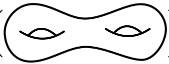
$\mathrm{Hom}(\Gamma, G) = ?$ $\quad\quad$ $\Gamma = \pi_1\Big($  $\Big).$

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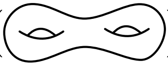
$\mathrm{Hom}(\Gamma, G) = ?$ $\Gamma = \pi_1\Big($  $\Big).$

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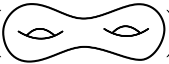
$\mathrm{Hom}(\Gamma, G) = ?$ $\qquad$ $\Gamma = \pi_1\Big($$\Big).$

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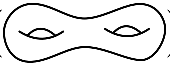
$\mathrm{Hom}(\Gamma, G) =?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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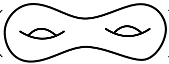
$\mathrm{Hom}(\Gamma, G) = ?$ $\qquad$ $\Gamma = \pi_1\Big($  $\Big).$

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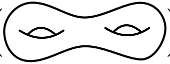
$\mathrm{Hom}(\Gamma, G) = ?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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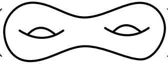
$\mathrm{Hom}(\Gamma, G) = ?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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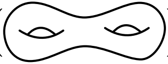
$\mathrm{Hom}(\Gamma, G) = ?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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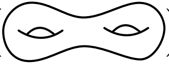
$\mathrm{Hom}(\Gamma, G) = ?$ $\Gamma = \pi_1\Big($  $\Big)$.

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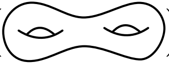
$\mathrm{Hom}(\Gamma, G) =?$ $\Gamma = \pi_1\Big($  $\Big)$.

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$$\mathscr{M}_\Gamma(G) = \mathrm{Hom}(\Gamma, G) /\!\!/ \mathrm{G}$$

(discard some ugly points).

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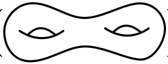
$\pi_0\Big(\mathscr{M}_\Gamma(G)\Big) = ?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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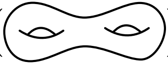
$\pi_0\Big(\mathscr{M}_\Gamma(G)\Big) = ?$ $\qquad \Gamma = \pi_1\Big($  $\Big).$

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G: non-compact real form.

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$$\tau(\rho_u) = \mathrm{Vol}(\Sigma) = 2\pi|\chi(\Sigma)|.$$

Milnor's Theorem: $\big|\tau(\rho)\big| \leq \mathrm{Vol}(\Sigma)$ for all ρ.

Goldman's Ph.D. thesis

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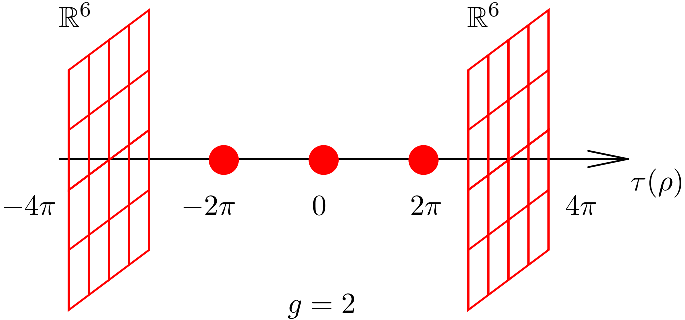

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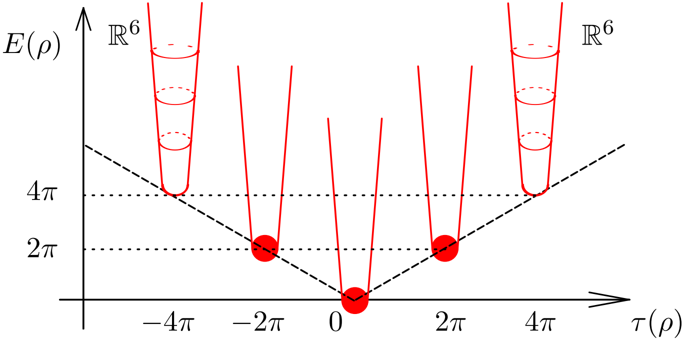

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- Critical points of $E(\rho) \iff$ "Hodge bundles".
- Morse indices of these points.

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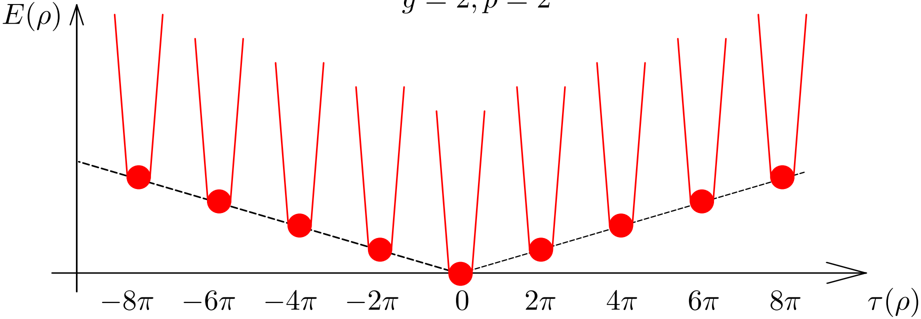

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Question: Are the minima there the only critical points? If so, description of the topology of those components (see S. 2014).

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Important example

Every cocompact torsion-free lattice $\Gamma < \mathrm{SU}(n,1)$. Indeed:

$$\mathbb{B}^n = \mathrm{SU}(n,1)/\mathrm{U}(n), \quad X = \Gamma \backslash \mathbb{B}^n \text{ is Kähler.}$$

Milnor–Wood inequality and maximality

We can still define

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Conjecture

If $|\tau(\rho)| = pn\mathrm{Vol}(X)$ then $G = \mathrm{SU}(p,q)$ and

$$\rho \sim \rho_{\mathrm{diag}} \colon \Gamma \subset \mathrm{SU}(1,n) \to \mathrm{SU}(p,np) \to \mathrm{SU}(p,q)$$

$$\begin{pmatrix} a & b \\ c & D \end{pmatrix} \mapsto \begin{pmatrix} a & & b & \\ & a & & b \\ c & & D & \\ & c & & D \end{pmatrix}$$

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Theorem (S. 2014)

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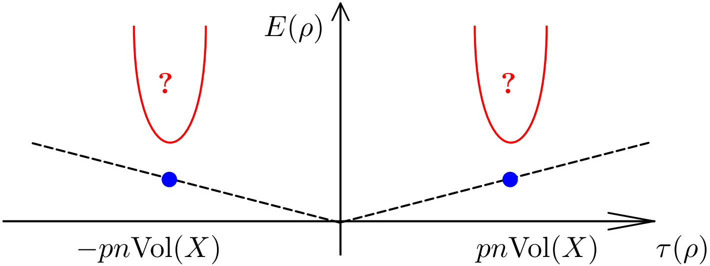

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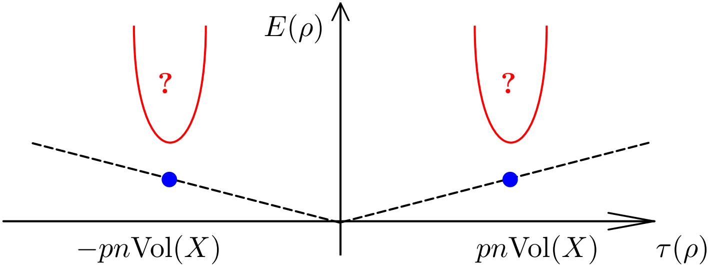

Conjecture (restated)

No 

components exist.

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Question: Can one adapt Bradlow–García-Prada–Gothen 2003 to higher dimension? (main techniques generalized in: S. 2013).
Problem: There is a counter-example. Kim–Klingler–Pansu 2009 constructed an isolated $\rho\colon \mathrm{SU}(1,n) \to \mathrm{SU}(2,n)$ such that $\tau(\rho) = 0, E(\rho) > 0$.

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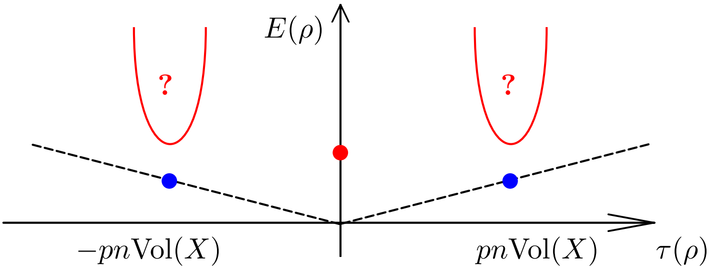
$(G = \mathrm{SU}(2, q), \mathrm{Sp}(4, \mathbb{R}), \mathrm{SO}_0(n, 2))$.
Question: Can one adapt Bradlow–García-Prada–Gothen 2003 to higher dimension? (main techniques generalized in: S. 2013).
Problem: There is a counter-example. Kim–Klingler–Pansu 2009 constructed an isolated $\rho\colon \mathrm{SU}(1, n) \to \mathrm{SU}(2, n)$ such that $\tau(\rho) = 0, E(\rho) > 0$.



Other things I am working on

- Goldman–Labourie conjecture on Teichmüller components and minimal maps;
- Harmonic maps and their dependency on the complex structure / Kähler metric;
- Complex variations of Hodge structure (from a combinatorial point of view?);
- Generalizations of harmonic maps and Corlette's theorem.